Development of 1st-order Particle Discretization Scheme (PDS) for analysis of cracking phenomena

Mahendra Kumar Pal[a], Lalith Wijerathne[b], Muneo Hori[c]

[a] Doctoral Student, Department of Civil Engineering, The University of Tokyo, Tokyo, Japan
[b] Associate Professor, Earthquake Research Institute, The University of Tokyo, Tokyo, Japan
[c] Professor, Earthquake Research Institute, The University of Tokyo, Tokyo, Japan

About me

Hello, I am Mahendra Kumar Pal, And I am pursuing Doctorate course in Department of Civil Engineering, the University of Tokyo.

I have received my B.Tech degree from B.I.E.T. Jhansi affiliated to UP technical University Lucknow in 2010, and M.Tech from Indian Institute of Technology, Hyderabad in 2012.

My research interests are computational solid mechanics, functional analysis, structural mechanics, earthquake engineering, mathematical modelling and numerical simulation.

Travelling, event planning and management are my favorite hobbies. I have explored the beautiful island called, Japan from its north-most Hokkaido, to Okinawa, the south most part of the land of rising sun.

Currently, I am the president of the University of Tokyo Indian Students' Association (UTISA).

Introduction

PDS approximate the function and derivative as union of local polynomial expansion about the mother point of conjugate domain tessellation, Φ^α, and Ψ^β.

Function approximation:

$$f^d(\boldsymbol{x}) = \sum_\alpha \left(\sum_n f^{\alpha n} P^n(\boldsymbol{x} - \boldsymbol{x}^\alpha) \varphi^\alpha(\boldsymbol{x}) \right)$$

Derivative approximation: over conjugate domain

$$g^d(\boldsymbol{x}) = \sum_\beta \left(\sum_n g^{\beta n} P^n(\boldsymbol{x} - \boldsymbol{x}^\beta) \psi^\beta(\boldsymbol{x}) \right)$$

For example: $P^4(\boldsymbol{x} - \boldsymbol{x}^\beta) = \{1, (x - x^\beta), (y - y^\beta), (z - z^\beta)\}$

Minimization of error norm

$$E = \int \left(f - f^d(x)\right)^2 dV \;\&\; E = \int \left(f_{,i}^d - g^d(x)\right)^2 dV$$

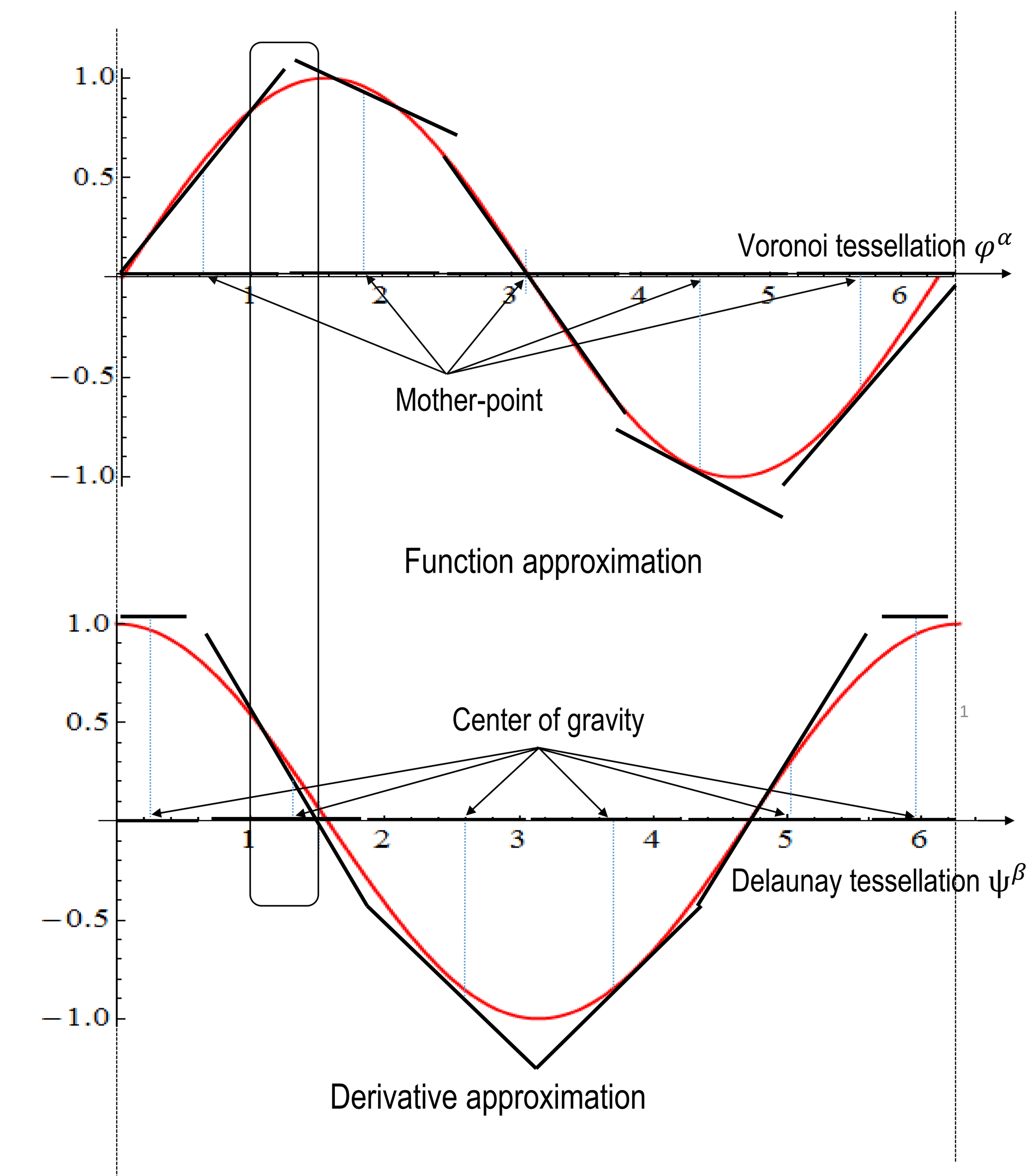
leads to

$$\sum_n I^{\alpha m n} f^{\alpha n} = \int P^m(\boldsymbol{x} - \boldsymbol{x}^\alpha)\varphi^\alpha(\boldsymbol{x}) f(\boldsymbol{x}) dV$$

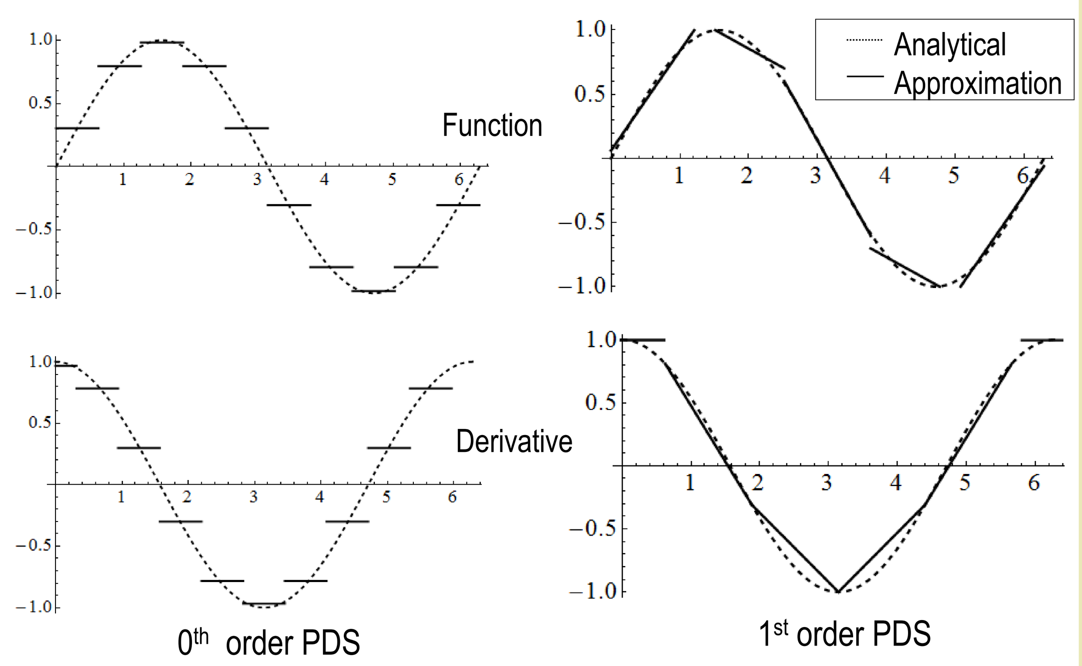
$$\sum_n I^{\beta m n} g_i^{\beta n} = \int P^m(\boldsymbol{x} - \boldsymbol{x}^\alpha)\psi^\beta(\boldsymbol{x}) f_{,i}^d(\boldsymbol{x}) dV$$

Where, $I^{\beta m n} = \int P^m(\boldsymbol{x} - \boldsymbol{x}^\beta) P^n(\boldsymbol{x} - \boldsymbol{x}^\beta)\psi^\beta(\boldsymbol{x}) dV$

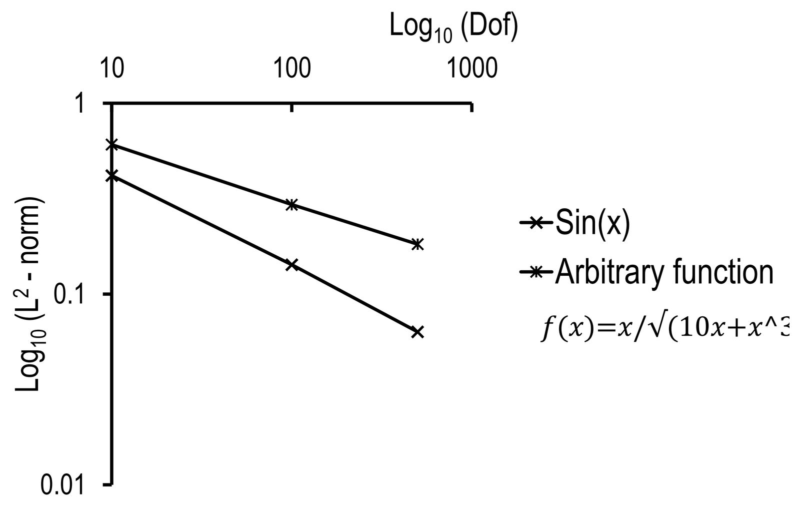
1st order PDS



Convergence rate 1st order PDS



Comparison between 0th order and 1st order PDS



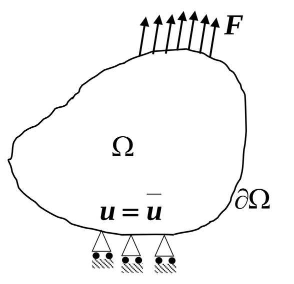
Convergence rate for derivative approximation

Governing equation

Consider the infinitesimal deformation of linear elastic body

$$\nabla.\boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{f} \quad \text{in } \Omega$$

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \quad \text{on } \partial\Omega$$



Body in static equilibrium

Lagrange for above governing equation

$$L[\boldsymbol{u}, \boldsymbol{\sigma}, \boldsymbol{\epsilon}] = \int \frac{1}{2}\boldsymbol{\epsilon}:\boldsymbol{c}:\boldsymbol{\epsilon} - \boldsymbol{\sigma}:(\boldsymbol{\epsilon} - \nabla\boldsymbol{u}) dV$$

The displacement vector, stress and strain fields are unknowns variable.

1st order PDS-FEM

For a static equilibrium problem, displacement vector, body force vector, stress and strain field are the variable

Displacement vector and body force approximation:

$$\boldsymbol{u}^d(\boldsymbol{x}) = \sum_\alpha \left(\sum_n \boldsymbol{u}^{\alpha n} P^n(\boldsymbol{x} - \boldsymbol{x}^\alpha)\varphi^\alpha(\boldsymbol{x}) \right)$$

Stress & strain field approximation:

$$\boldsymbol{\epsilon}^d(\boldsymbol{x}) = \sum_\beta \left(\sum_n \boldsymbol{\epsilon}^{\beta n} P^n(\boldsymbol{x} - \boldsymbol{x}^\beta)\psi^\beta(\boldsymbol{x}) \right)$$

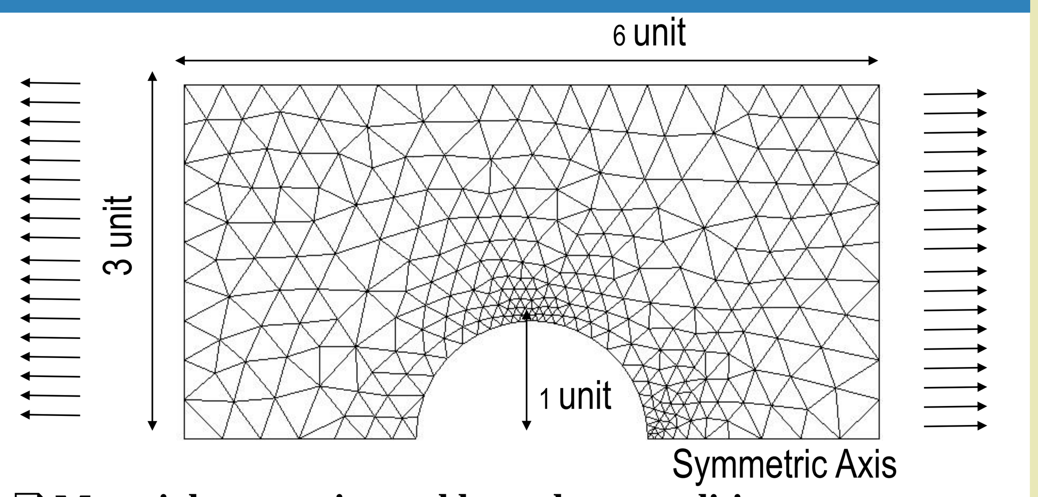
$$\boldsymbol{\sigma}^d(\boldsymbol{x}) = \sum_\beta \left(\sum_n \boldsymbol{\sigma}^{\beta n} P^n(\boldsymbol{x} - \boldsymbol{x}^\beta)\psi^\beta(\boldsymbol{x}) \right)$$

$\{\boldsymbol{u}^{\alpha n}\}, \{\boldsymbol{\epsilon}^{\beta n}\}$ and $\{\boldsymbol{\sigma}^{\beta n}\}$ are the set of unknowns

In order to investigate the efficiency of the formulation: $\{u_x, u_y\}$ and $\{v_x, v_y\}$ has been calculated from the analytical solution

$$v = \begin{Bmatrix} v^{\alpha_0} \\ v^{\alpha_1} \\ v^{\alpha_2} \end{Bmatrix} = \begin{Bmatrix} v^{\alpha_0} \\ v^{\alpha_1} \\ 0 \end{Bmatrix} \quad u = \begin{Bmatrix} u^{\alpha_0} \\ u^{\alpha_1} \\ u^{\alpha_2} \end{Bmatrix} = \begin{Bmatrix} u^{\alpha_0} \\ u^{\alpha_1} \\ 0 \end{Bmatrix}$$

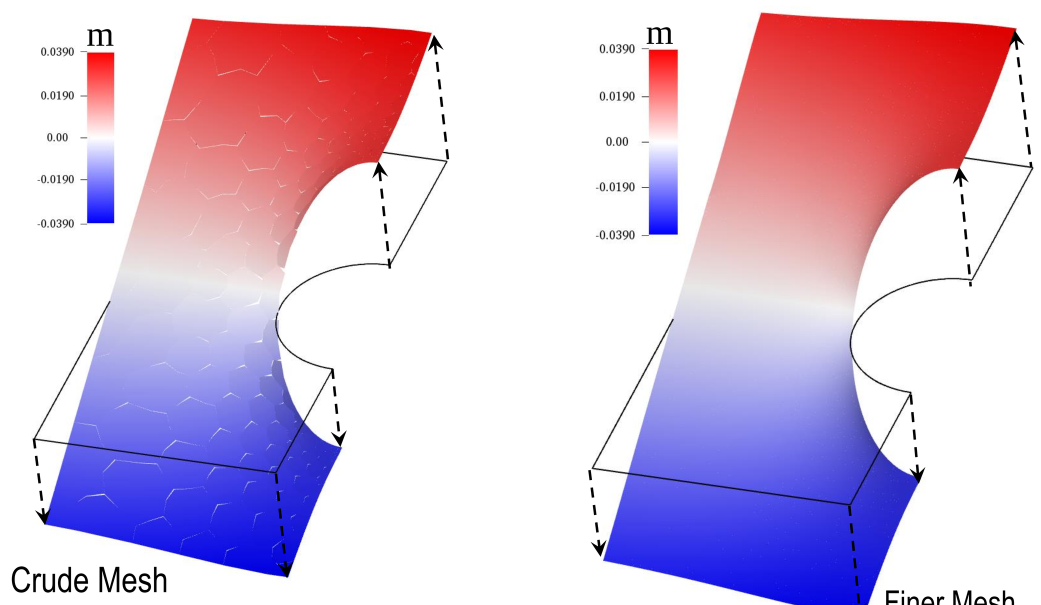
Numerical example and results



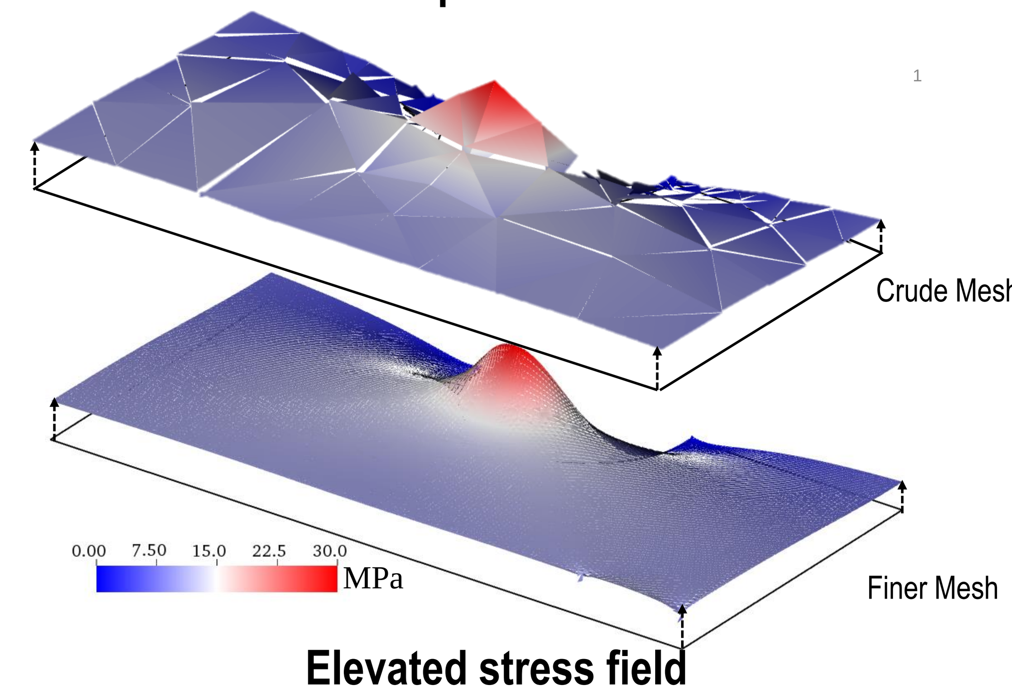
Material properties and boundary conditions

- Young's Modulus = 1000 MPa, Poisson's ratio = 0.33
- Stress far field =10 MPa

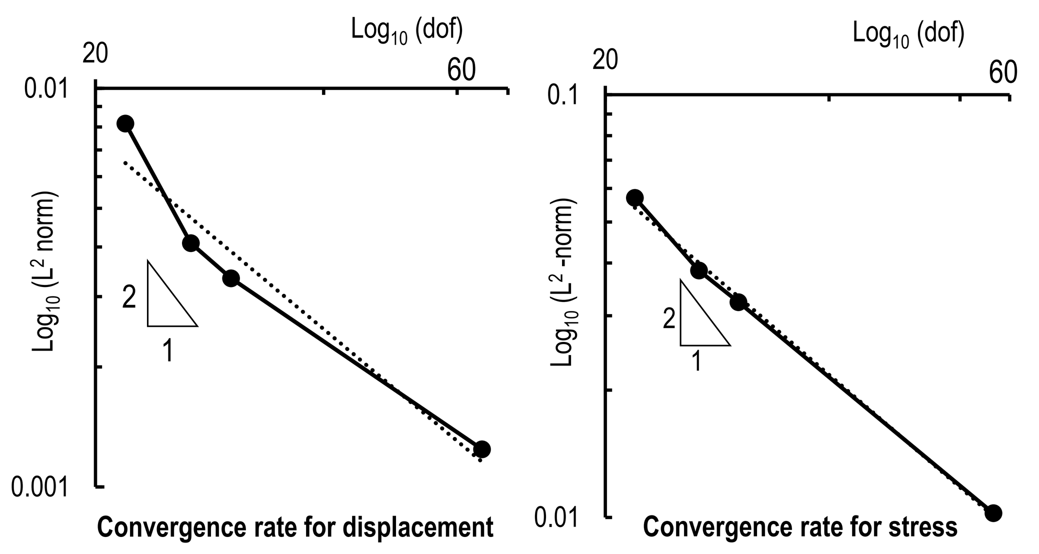
Numerical Example



Elevated displacement field



Elevated stress field



Summary

- 1st order PDS-FEM is proposed and implemented for 2D problems
- 2nd order convergence rate is obtained by approximating the unknown variables with first order polynomial series expansion

Future work

- Efficient treatment of boundary conditions
- Implementing techniques for modeling propagating cracks
- Implementation of 3D PDS-FEM

Contact address: mahendra@eri.u-tokyo.ac.jp